We study Distributed Estimation (DES) problem in Wireless Sensor Network (WSN), where several battery powered geographically distributed tiny sensors observe a noisy version of an underlying unknown physical phenomena, and transmit a compressed version of their observations over erroneous communication channels to a Fusion Center (FC), where collective data is fused to reconstruct the unknown with minimum Mean Square Error (MSE). The accuracy of DES depends on many factors such as intensity of observation noises (include additive and multiplicative noises) in sensors, quantization errors in sensors, available power and bandwidth of the network, quality of communication channels between sensors and the FC, and the choice of fusion rule in the FC. Taking into account all of these contributing factors and implementing a DES system which minimizes the MSE and satisfies all constraints is a challenging task. We consider an inhomogeneous WSN where the sensors' observations are modeled linear with additive Gaussian noise. The communication channels between sensors and FC are orthogonal bandwidth-constrained erroneous wireless channels. The unknown to be estimated is a Gaussian. Sensors employ uniform multi-bit quantizers and digital modulation. We find the best fusion rule in the FC and best transmit power and quantization rate (measured in bits per sensor) allocation schemes that minimize the MSE. Investigating the proposed approaches helps us to scrutinize the effect of contributing factors on the MSE. In sequel we dig into role of multiplicative noise on DES system performance and its effect on resource allocation for sensors. As a dual problem we also answer the question: what is the minimum required bandwidth of the network to satisfy a predetermined target MSE? We also derive Bayesian Cramer-Rao Lower Bound (BCRLB) and compare the MSE performance of our proposed methods against the bound.