This thesis describes the development, verification, and validation of a three-dimensional time-domain thermoacoustic solver. The purpose of the solver is to predict the frequencies, modeshapes, linear growth rates, and limit cycle amplitudes for combustion instability modes in gas turbine combustion chambers. The treatment of mean density gradients was found to be vital to the success of frequency and modeshape predictions due to the sharp density gradients that occur across deflagration waves. In order to treat mean density gradients with physical fidelity, a non-conservative finite volume method based on the wave propagation approach to the Riemann problem is applied. For modelling unsteady heat release, user input flexibility is maximized using a virtual class hierarchy within the OpenFOAM C++ library. Unsteady heat release based on time lag models are demonstrated. The solver gives accurate solutions compared with analytical methods for one-dimensional cases involving mean density gradients, cross-sectional area changes, uniform mean flow, arbitrary impedance boundary conditions, and unsteady heat release in a one-dimensional Rijke tube. The solver predicted resonant frequencies within 1% of the analytical solution for these verification cases, with the dominant component of the error coming from the finite time interval over which the simulation is performed. The linear growth rates predicted by the solver for the Rijke tube verification were within 5% of the theoretical values, provided that numerical dissipation effects were controlled. Finally, the solver is then used to predict the frequencies and limit cycle amplitudes for two lab scale experiments in which detailed acoustics data are available for comparison. For experiments at the University of Melbourne, an empirical flame describing function was provided. The present simulation code predicted a limit cycle of 0.21 times the mean pressure, which was in close agreement with the estimate of 0.25 from the experimental data. The experiments at Purdue University do not yet have an empirical flame model, so a general vortex-shedding model is proposed on physical grounds. It is shown that the coefficients of the model can be tuned to match the limit cycle amplitude of the 2L mode from the experiment with the same accuracy as the Melbourne case. The code did not predict the excitation of the 4L mode, therefore it is concluded that the vortex-shedding model is not sufficient and must be supplemented with additional heat release models to capture the entirety of the physics for this experiment.